Math 180, Quiz 8: Thursday, March 21, 2013

Take fifteen minutes to complete this quiz. Please show all your work, and write your name on the front *and* back of the paper before turning it in. Make sure to show all relevant work.

1. **(9 pts)** Sketch a graph of the function $f(x) = \dfrac{x^2 - 5}{x - 3}$. In particular:

   - Make sure your sketch is increasing and decreasing in the right places and displays the appropriate concavity and end behavior.
   - Indicate any critical and/or inflection points on the graph, including their coordinates.
   - Indicate any asymptotes with dashed lines.
   - Write down all calculations needed to establish the features above.

   (They probably won't give you this checklist on the exam, but I thought I'd be nice and add it here.)

$$f'(x) = \frac{(x-3)(2x) - (x^2-5)(1)}{(x-3)^2} = \frac{2x^2 - 6x - x^2 + 5}{(x-3)^2} = \frac{x^2 - 6x + 5}{(x-3)^2} = \frac{(x-5)(x-1)}{(x-3)^2}$$

$$f''(x) = \frac{(x-3)^2(2x-6) - (x^2-6x+5)2(x-3)}{(x-3)^4 \, 3} = \frac{(2x^2-12x+18) - (2x^2-12x+10)}{(x-3)^3} = \frac{8}{(x-3)^3}$$

f' $\oplus$ $\ominus$ $\ominus$ $\oplus$ →   (number line: 1, 3, 5)
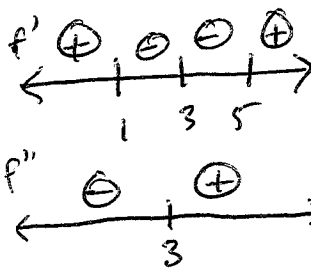
$$f(1) = \frac{1-5}{1-3} = \frac{-4}{-2} = 2 \quad \text{(local max)}$$

$$f(5) = \frac{25-5}{5-3} = \frac{20}{2} = 10 \quad \text{(local min)}$$

$$\lim_{x\to 3^+} \frac{\overbrace{x^2-5}^{\to 4}}{\underbrace{x-3}_{\to 0,\ \text{positive}}} = +\infty \quad \Big\} \text{ VA}$$

$$\lim_{x\to 3^-} \frac{\overbrace{x^2-5}^{\to 4}}{\underbrace{x-3}_{\to 0,\ \text{negative}}} = -\infty \quad \Big\} \; x=3$$

f'' $\ominus$ $\oplus$ →   (number line: 3)

$$\lim_{x\to\infty} \frac{x^2-5}{x-3} = \lim_{x\to\infty} \frac{x(x - 5/x)}{x(1 - 3/x)} = \lim_{x\to\infty} \frac{\overbrace{x - 5/x}^{\to\infty}}{\underbrace{(1 - 3/x)}_{\to 1}} = +\infty$$

$$\lim_{x\to-\infty} \frac{x^2-5}{x-3} = \lim_{x\to-\infty} \frac{\overbrace{x - 5/x}^{\to-\infty}}{\underbrace{(-3/x)}_{\to 1}} = -\infty$$

(graph with points $(1,2)$ and $(5,10)$, vertical asymptote at 3)
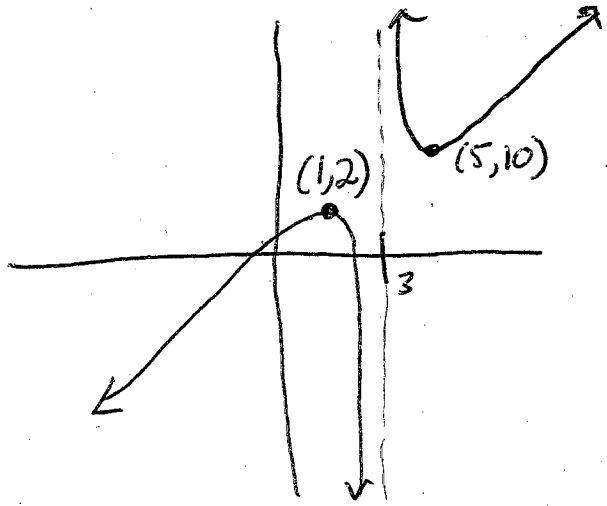
2. **(1 pt)** Find the absolute extrema of the function $f(x)$ above on the interval $[-1, 2]$.

$$f(-1) = \frac{1-5}{-1-3} = \frac{-4}{-4} = 1 \quad \longleftarrow$$

$$\text{(from above) } f(1) = 2 \quad \longleftarrow \text{ absolute maximum}$$

$$f(2) = \frac{4-5}{2-3} = \frac{-1}{-1} = 1 \quad \longleftarrow$$

absolute minima